

Name: _____

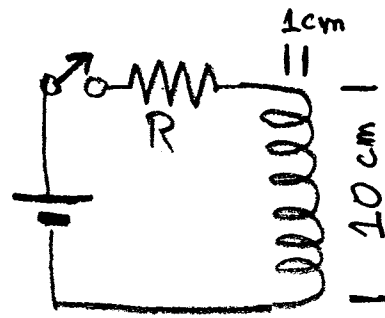
Perm: Avg = 54.5/60 = 90.8%

This exam is 50min long and closed book. You may use a single formula sheet and a calculator. You may not, of course, get any help from any other person. The test consists of three problems, of equal weight. Write your answers on this test itself, or on scratch paper. Please show your work, and turn all your work in.

Good luck!

1. (20 pt) *An inductor.* An inductor is made as a solenoid with 1000 turns of wire, with a circular cross-section of radius 1cm, and a length of 10cm.

- What is its inductance L ?
- The inductor is now connected in series with a 9V battery and a 10 ohm resistor. What is the time constant of this circuit?
- After a long time, how much energy is stored in the inductor?



a) $L = \mu_0 n^2 A l$ $\mathcal{E} = 9V$

$n = \frac{1000 \text{ turn}}{0.1 \text{ m}} = 10^5 \frac{\text{turn}}{\text{m}}$

$A = \pi r^2 = \pi (0.01 \text{ m})^2$ so $L = 3.95 \times 10^{-3} \text{ H}$

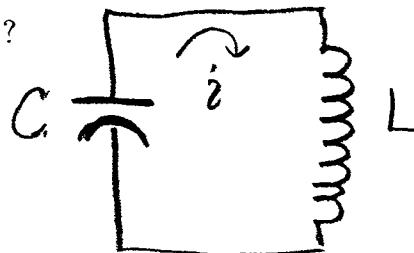
b) $\tau_L = L/R = \underline{3.95 \times 10^{-4} \text{ s}}$

c) After a long time (many τ_L) the current is $i = \mathcal{E}/R = 0.9 \text{ A}$ (because $di/dt = 0$ so the inductor has no \mathcal{E}_L across it.)

$U_L = \frac{1}{2} L i^2 = \underline{1.60 \times 10^{-3} \text{ J}}$

2. (20 pt) An AC circuit. An inductor $L = 0.01$ H and a capacitor $C = 0.0001$ F are connected in series. A current of maximum value $i_m = 20$ A is set flowing in the circuit.

- What is the resonant frequency in Hertz (cycles per second)?
- What is the maximum EMF across the inductor?
- What is the maximum energy stored in the inductor?
- What is the maximum energy stored in the capacitor?



a) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2} \cdot 10^{-4}}} = 10^3$ rad/sec

b) $E_L = X_L i$ where $X_L = \omega L = 10^3 \cdot 10^{-2} = 10$.

$E_{L,m} = 10 \cdot 20$ $E_{L,m} = 200$ V

c) $U_{L,max} = \frac{1}{2} L i_m^2 = \frac{1}{2} \cdot 0.01 \cdot (20)^2 = 2$ J

d) There is no energy dissipation because $R=0$. Therefore energy cycles back and forth between the inductor and the capacitor, with no loss; so

$U_{C,max} = U_{L,max} = 2$ J

3. (20 pt) An electromagnetic wave. The electric field of a wave with maximum value $E_m = 10^5$ V/m is

$$\vec{E}(y, t) = E_m \hat{i} \sin(ky - \omega t)$$

(here $\hat{i}, \hat{j}, \hat{k}$ are unit vectors which point in directions x, y, z , respectively).

- In what direction is the wave propagating?
- Write a corresponding equation for the magnetic field $\vec{B}(y, t)$. What is the numerical maximum value B_m ? In what direction does \vec{B} point?
- Write an equation for the Poynting vector of this wave.
- What is the average intensity of this wave?

a) Because the $\sin()$ depends on y , not any other spatial coordinate, the wave must propagate along y , or \hat{j} . $\frac{dy}{dt} = \frac{\omega}{k}$.

b) Since \vec{E} points along \hat{i} , and the wave propagates along \hat{j} , \vec{B} must point along \hat{k} or maybe $-\hat{k}$. Because $\vec{E}, \vec{B}, \hat{j}$ must be right-handed, \vec{B} points along $-\hat{k}$ and

$$\vec{B}(y, t) = B_m (-\hat{k}) \sin(ky - \omega t), B_m = \frac{E_m}{c} = 3.33 \times 10^{-4}$$

$$c) \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \left(\frac{E_m^2}{\mu_0 c} \right) \hat{j} \sin^2(ky - \omega t)$$

$$d) I_{avg} = \frac{S_{max}}{2} = \frac{E_m^2}{2\mu_0 c} = 1.33 \times 10^7 \text{ W/m}^2$$

End of Test