UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 221A **Quantum Field Theory** **Fall 2007**

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http://www.kitp.ucsb.edu/~joep/Web221A/221A.html

FINAL EXAM

Open notes, homework, solutions, Srednicki (the text, not the person). Please do not discuss the test with anyone but me before the due time. I will check my email regularly, and post any corrections/clarifications on the course web page.

Begin: Tuesday, Dec. 11, noon.
Due: Wednesday, Dec. 12, noon in my office, KITP 2319 (except for those who have made other arrangements).

1. Consider a theory in $d = 4$ with three real scalar fields A, B, and C, and the following Lagrangian density:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu A \partial^\mu A + \partial_\mu B \partial^\mu B + \partial_\mu C \partial^\mu C + m_A^2 A^2 + m_B^2 B^2 + m_C^2 C^2) - \frac{g}{2} A B^2 - \frac{h}{2} C B^2 \ .$$

There might be other terms in $\mathcal{L}$ but you won't need them (in particular you won't need counterterms).

a) Write the Feynman rules for this theory (propagators and vertices).

b) Consider first the case $m_A > 2m_B$. What is the tree level decay rate $A \to B + B$?

c) Now consider the case $m_A < 2m_B$ but $m_A > 2m_C$. Show that the decay $A \to C + C$ can happen at one loop.

d) Evaluate the rate $A \to C + C$ at one loop. You don't need to do the Feynman parameter integral in closed form, but do the loop momentum integral and leave the answer in terms of a Feynman parameter integral expressed as a function of m_A^2, m_B^2, and m_C^2.

e) For $m_B^2 \gg m_A^2$ and $m_B^2 \gg m_C^2$, evaluate the Feynman parameter integral. The rate in this limit is equivalent to that you would get in a field theory without the B particle but with a vertex $\tilde{g} A C^2$. Evaluate $\tilde{g}$.

f) Show that it is impossible for B to decay, no matter how heavy it is, with only the indicated couplings.

2. In the last homework you derived the most general theory with two scalar fields and certain discrete symmetries, which we can write as

$$\mathcal{L} = -\frac{Z_\phi}{2}(\partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \phi_2 \partial^\mu \phi_2) - \frac{Z_g}{4!} g (\phi_1^4 + \phi_2^4) - \frac{Z_\lambda}{4!} \lambda \phi_1^2 \phi_2^2 \ .$$

For simplicity I have set the mass to zero.

a) Evaluate the one-loop β-functions, in $d = 4$, for g and λ.

b) Write the renormalization group equation for the ratio λ/g. Given initial values g and λ at some scale μ, what value does the ratio approach at low energy? (This may depend on the initial values, but only a discrete set of final ratios is possible). The condition for stability from the last homework was $g > 0$ and $\lambda > -2g$; you may assume this for your initial values.

3. Consider a theory with $SO(3)$ symmetry and *two* triplets of fields, ϕ_i and χ_i, where i runs from 1 to 3. Let $\mathcal{L}$ be

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi_i \partial^\mu \phi_i + \partial_\mu \chi_i \partial^\mu \chi_i + m^2 \phi_i \phi_i + m^2 \chi_i \chi_i)$$
$$-\frac{g}{8}(\phi_i \phi_i)(\phi_j \phi_j) - \frac{g}{8}(\chi_i \chi_i)(\chi_j \chi_j) - \frac{\lambda}{2}(\phi_i \chi_i)(\phi_j \chi_j) \ .$$

a) For $m^2 < 0$, find the vacua. The answer will depend on the sign of λ (you can assume that λ is not too negative, so that the potential is bounded below).

b) Find the unbroken symmetry, both for $\lambda > 0$ and $\lambda < 0$.

c) By shifting the fields to expand around the new minimum, look at the mass matrix and verify that you have the correct number of Goldstone bosons.