UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 221A **Quantum Field Theory** **Fall 2007**

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ASSIGNMENT #4

Due: Friday, Oct. 26, 5pm in TA's mailbox (5th floor Broida). See course web page for late homework policy.

1. Srednicki 9.3

2. (A simpler version of 9.4). Consider the convergent integral

$$Z_\lambda = \int_{-\infty}^{\infty} dx \, \exp\left[-\frac{1}{2}x^2 - \frac{\lambda}{4!}x^4\right] \; .$$

This is a toy model of QFT, with spacetime reduced to a point. We could include a source Jx, but I will keep it simple.

a) Expand the integrand in powers of λ and calculate the general term. Show that the radius of convergence is zero.

Although it is not convergent, this expansion has a well-defined meaning as an asymptotic series. This means that if we define $Z_{n,\lambda}$ as the sum of the terms through $O(\lambda^n)$, then

$$\lim_{\lambda\to 0}(Z_\lambda - Z_{n,\lambda})/\lambda^n = 0 \; .$$

That is, the error is smaller that the last term retained, as $\lambda \to 0$. This toy model captures the result that QFT perturbation theory is generally asymptotic but not convergent. Again, convergence refers to summing all terms at fixed λ, asymptotic refers to keeping any finite number of terms and taking $\lambda \to 0$.

b) The Feynman rules are propagator = 1, four-point vertex = $-\lambda$, so the only nontrivial part is the symmetry factors. It can be shown that Z_λ/Z_0 is equal to the sum of *vacuum* graphs. Verify this explicitly at orders $\lambda^{1,2,3}$.

3. Srednicki 10.3

4. Srednicki 10.4

5. Srednicki 10.5

I am not assigning 9.5 and 10.1. These are how perturbation theory was done before path integrals. You might have a look at these, because the methods are still sometimes useful.